Sean las matrices

$$A = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & a & 1 \\ b & c & -1 \end{pmatrix} \qquad C = \begin{pmatrix} -1 & 1 & -1 \\ -2 & 0 & -2 \end{pmatrix}$$

a) [0,75] Halle los valores de a, b y c para que se verifique $B \cdot C^t + 4A = O$.

b) [1,5] Resuelva la ecuación matricial $X \cdot A - A^2 = 3\mathrm{I}$.

c) [0,75] Obtén razonadamente A^{2016} y A^{2017}.

ⓐ $B \cdot C^t = \begin{pmatrix} 1 & a & 1 \\ b & c & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 \\ 1 & 0 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} a-2 & -4 \\ c-b+1 & -2b+2 \end{pmatrix}$

$4A = \begin{pmatrix} -4 & 4 \\ 0 & -4 \end{pmatrix}$

$B C^t + 4A = \begin{pmatrix} a-6 & 0 \\ c-b+1 & -2b-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$\rightarrow \begin{cases} a-6=0 \longrightarrow \boxed{a=6} \\ 0 = 0 \checkmark \\ c-b+1 = 0 \longrightarrow c = b-1 \rightarrow c = -1-1 \rightarrow \boxed{c=-2} \\ -2b-2=0 \longrightarrow -2b = 2 \rightarrow b = \frac{2}{-2} \rightarrow \boxed{b=-1} \end{cases}$

$A = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

ⓑ $X \cdot A - A^2 = 3I \longrightarrow X \cdot A = 3I + A^2 \longrightarrow X = (3I + A^2) \cdot A^{-1}$

$A^2 = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

$3I = 3 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

$3I + A^2 = \begin{pmatrix} 4 & -2 \\ 0 & 4 \end{pmatrix} \leftarrow$

$\det(A) = (-1)(-1) - 0 \cdot 1 = 1$

$A_{11} = (-1)^2 \cdot (-1) = -1$	$A_{12} = (-1)^3 \cdot 0 = 0$
$A_{21} = (-1)^3 \cdot 1 = -1$	$A_{22} = (-1)^4 \cdot (-1) = -1$

$A^{-1} = \frac{1}{\det(A)} \cdot Adj(A)^t = \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \leftarrow$

$$X = \begin{pmatrix} 4 & -2 \\ 0 & 4 \end{pmatrix} \cdot \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix} \Rightarrow \boxed{X = \begin{pmatrix} -4 & -2 \\ 0 & -4 \end{pmatrix}}$$

c) $A = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 3 \\ 0 & -1 \end{pmatrix}$$

$$A^4 = A^3 \cdot A = \begin{pmatrix} -1 & 3 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -4 \\ 0 & 1 \end{pmatrix}$$

$$A^5 = \{\text{Vemos que es}\} = \begin{pmatrix} -1 & 5 \\ 0 & -1 \end{pmatrix}$$

n impar $\longrightarrow A^n = \begin{pmatrix} -1 & n \\ 0 & -1 \end{pmatrix} \Rightarrow A^{2017} = \begin{pmatrix} -1 & 2017 \\ 0 & -1 \end{pmatrix}$

n par $\longrightarrow A^n = \begin{pmatrix} 1 & -n \\ 0 & 1 \end{pmatrix} \Rightarrow A^{2016} = \begin{pmatrix} 1 & -2016 \\ 0 & 1 \end{pmatrix}$

Por induuuuuccción

EJERCICIO 3

Sea la matriz

$$E = \begin{pmatrix} 7 & 1 & -2 \\ 0 & 0 & 0 \\ 28 & 4 & -8 \end{pmatrix}$$

a) [0,75] Determine para qué valores de λ tiene inversa la matriz $\boxed{F = \lambda I - E}$.

b) [1] Obtenga la inversa de F para $\lambda = 1$.

c) [0,75] Estudie el rango de F según los valores de λ.

ⓐ

$$\lambda \cdot I = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \Rightarrow F = \lambda \cdot I - E = \begin{bmatrix} \lambda-7 & -1 & 2 \\ 0 & \lambda & 0 \\ -28 & -4 & \lambda+8 \end{bmatrix}$$

$$\det(F) = \lambda \cdot (\lambda^2 + 8\lambda - 7\lambda - 56) + 56\lambda = \lambda(\lambda^2 + \lambda) = \lambda^2 \cdot (\lambda + 1)$$

$\det(F) = 0$:
- $\lambda^2 = 0 \rightarrow \lambda = 0$
- $\lambda + 1 = 0 \rightarrow \lambda = -1$

Aclaramos: Si $\lambda = 0$ ó $\lambda = -1 \Rightarrow det(F) = 0 \Rightarrow F$ no tiene inversa

Si $\lambda \neq 0$ y $\lambda \neq -1 \Rightarrow det(F) \neq 0 \Rightarrow F$ sí tiene inversa

(b) $\lambda = 1 \longrightarrow F = \begin{bmatrix} -6 & -1 & 2 \\ 0 & 1 & 0 \\ -28 & -4 & 9 \end{bmatrix} \longrightarrow F^{-1} = \frac{1}{det(F)} \cdot Adj(F)^t = \frac{1}{2}\begin{bmatrix} \ & \ & \ \\ \ & \ & \ \\ \ & \ & \ \end{bmatrix}$

$det(F) = 1^2 \cdot (1+1) = 1 \cdot 2 = 2$

$F_{11} = (-1)^2 \cdot \begin{vmatrix} \ \end{vmatrix}$ $\quad F_{12} = (-1)^3 \begin{vmatrix} \ \end{vmatrix}$ $\quad F_{13} = (-1)^4 \begin{vmatrix} \ \end{vmatrix}$

$F_{21} = (-1)^3 \begin{vmatrix} \ \end{vmatrix}$ $\quad F_{22} = (-1)^4 \begin{vmatrix} \ \end{vmatrix}$ $\quad F_{23} = (-1)^5 \begin{vmatrix} \ \end{vmatrix}$

$F_{31} = (-1)^4 \begin{vmatrix} \ \end{vmatrix}$ $\quad F_{32} = (-1)^5 \begin{vmatrix} \ \end{vmatrix}$ $\quad F_{33} = (-1)^6 \begin{vmatrix} \ \end{vmatrix}$