Sea la matriz

$$E = \begin{pmatrix} 7 & 1 & -2 \\ 0 & 0 & 0 \\ 28 & 4 & -8 \end{pmatrix}$$

a) [0,75] Determine para qué valores de λ tiene inversa la matriz $F = \lambda \mathrm{I} - E$.

b) [1] Obtenga la inversa de F para $\lambda = 1$.

c) [0,75] Estudie el rango de F según los valores de λ.

Bollullos 24 febrero 2021.

a) Calculamos F.

$$F = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} - E = \boxed{\begin{pmatrix} \lambda-7 & -1 & +2 \\ 0 & \lambda & 0 \\ -28 & -4 & \lambda+8 \end{pmatrix}} \leftarrow F$$

Calculamos $\det(F) = \lambda(\lambda^2 + 8\lambda - 7\lambda - 56) + 56\lambda = \lambda(\lambda^2 + \lambda) = \lambda^2(\lambda + 1)$

Igualamos a cero:

$$\lambda^2(\lambda+1) = 0 \begin{cases} \lambda^2 = 0 \rightarrow \lambda = 0 \\ \lambda + 1 = 0 \rightarrow \lambda = -1 \end{cases}$$

Aclaramos:

Si $\lambda = 0$ ó $\lambda = -1 \Rightarrow \det(F) = 0 \Rightarrow$ F no tiene inversa.

Si $\lambda \neq 0$ y $\lambda \neq -1 \Rightarrow \det(F) \neq 0 \Rightarrow$ F sí tiene inversa.

b) $\lambda = 1 \rightarrow$

$$F = \begin{pmatrix} -6 & -1 & 2 \\ 0 & 1 & 0 \\ -28 & -4 & 9 \end{pmatrix}$$

$$F^{-1} = \frac{1}{\det(F)} \cdot \mathrm{Adj}(F)^t =$$

$\det(F) = 1^2 \cdot (1+1) = 2$

$$= \frac{1}{2} \begin{pmatrix} 9 & 1 & -2 \\ 0 & 2 & 0 \\ 28 & 4 & -6 \end{pmatrix}$$

$$A_{11} = \begin{vmatrix} 1 & 0 \\ -4 & 9 \end{vmatrix} = 9 \qquad A_{12} = -\begin{vmatrix} 0 & 0 \\ * & * \end{vmatrix} = 0 \qquad A_{13} = \begin{vmatrix} 0 & 1 \\ -28 & -4 \end{vmatrix} = +28$$

$$A_{21} = -\begin{vmatrix} -1 & 2 \\ -4 & 9 \end{vmatrix} = 1 \qquad A_{22} = \begin{vmatrix} -6 & 2 \\ -28 & 9 \end{vmatrix} = 2 \qquad A_{23} = -\begin{vmatrix} -6 & -1 \\ -28 & -4 \end{vmatrix} = 4$$

$$A_{31} = \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} = -2 \qquad A_{32} = -\begin{vmatrix} -6 & 2 \\ 0 & 0 \end{vmatrix} = 0 \qquad A_{33} = \begin{vmatrix} -6 & -1 \\ 0 & 1 \end{vmatrix} = -6$$

c)

$$F = \begin{bmatrix} \lambda-7 & -1 & 2 \\ 0 & \lambda & 0 \\ -28 & -4 & \lambda+8 \end{bmatrix} \qquad \det(F) = \lambda^2(\lambda+1) = 0 \begin{cases} \lambda = 0 \\ \lambda = -1 \end{cases}$$

ⓐ Si $\lambda \neq 0$ y $\lambda \neq -1 \Rightarrow \Delta_3 = \det(F) \neq 0 \Rightarrow rg(F) = 3$

ⓑ Si $\boxed{\lambda = 0} \longrightarrow \Delta_3 = \det(F) = 0$

$$F = \begin{bmatrix} -7 & -1 & 2 \\ 0 & 0 & 0 \\ -28 & -4 & 8 \end{bmatrix} \longrightarrow \text{Todos los } \Delta_2 = 0$$

$\left.\begin{matrix} \Delta_3 = \det(F) = 0 \\ \text{Todos los } \Delta_2 = 0 \end{matrix}\right\} rg(F) = 1$

ⓒ Si $\lambda = -1 \longrightarrow \Delta_3 = \det(F) = 0$

$$F = \begin{bmatrix} -8 & -1 & 2 \\ 0 & -1 & 0 \\ -28 & -4 & 7 \end{bmatrix} \longrightarrow \text{Es } \Delta_2 = \begin{vmatrix} -8 & -1 \\ 0 & -1 \end{vmatrix} \neq 0$$

$\left.\begin{matrix} \Delta_3 = \det(F) = 0 \\ \Delta_2 \neq 0 \end{matrix}\right\} rg(F) = 2$