Examen 2 - Ejercicio 3 (Propiedades determinantes).

$$M = \begin{pmatrix} 1 & 2 & 3 \\ 6 & 0 & 3 \\ x & y & z \end{pmatrix} \qquad \det(M) = 2$$

ⓐ $M \cdot M^{-1} = I \xrightarrow{(*)} \underbrace{\det(M)}_{2} \cdot \det(M^{-1}) = \underbrace{\det(I)}_{1} \rightarrow \det(M^{-1}) = \dfrac{1}{2}$

ⓑ $\begin{vmatrix} 2 & 0 & 1 \\ 1 & 2 & 3 \\ x & y & z \end{vmatrix} \underset{f_2 \rightleftarrows f_1}{\overset{(*)}{=}} - \begin{vmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ x & y & z \end{vmatrix} = - \begin{vmatrix} 1 & 2 & 3 \\ 6/3 & 0/3 & 3/3 \\ x & y & z \end{vmatrix} \overset{(*)}{=} -\dfrac{1}{3} \cdot \begin{vmatrix} 1 & 2 & 3 \\ 6 & 0 & 3 \\ x & y & z \end{vmatrix} = -\dfrac{1}{3} \cdot 2 = \dfrac{-2}{3}$

ⓒ $\begin{vmatrix} 1 & x+6 & x \\ 2 & y & y \\ 3 & z+3 & z \end{vmatrix} \overset{(*)}{=} \begin{vmatrix} 1 & 2 & 3 \\ x+6 & y & z+3 \\ x & y & z \end{vmatrix} \underset{f_2 - f_3}{\overset{(*)}{=}} \begin{vmatrix} 1 & 2 & 3 \\ 6 & 0 & 3 \\ x & y & z \end{vmatrix} = 2$

ⓓ $N^2 \cdot M \cdot N = 3M^t \longrightarrow |N| \cdot |N| \cdot \underbrace{|M|}_{2} \cdot |N| = \underbrace{|3M^t|}_{54}$

$3 \cdot M^t = \begin{pmatrix} 3 & 18 & 3x \\ 6 & 0 & 3y \\ 9 & 9 & 3z \end{pmatrix} \Rightarrow |3M^t| = \begin{vmatrix} 3 & 18 & 3x \\ 6 & 0 & 3y \\ 9 & 9 & 3z \end{vmatrix} \overset{(*)}{=} 3 \cdot 3 \cdot 3 \cdot \begin{vmatrix} 1 & 6 & x \\ 2 & 0 & y \\ 3 & 3 & z \end{vmatrix} \overset{(*)}{=} 3 \cdot 3 \cdot 3 \cdot 2 = 54$

$\longrightarrow |N|^3 = \dfrac{54}{2} \longrightarrow |N|^3 = 27 \longrightarrow |N| = \sqrt[3]{27} \longrightarrow |N| = 3$

Sean las matrices

$$A = \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & a & 1 \\ b & c & -1 \end{pmatrix} \qquad C = \begin{pmatrix} -1 & 1 & -1 \\ -2 & 0 & -2 \end{pmatrix}$$

a) [0,75] Halle los valores de a, b y c para que se verifique $B \cdot C^t + 4A = O$.

b) [1,5] Resuelva la ecuación matricial $X \cdot A - A^2 = 3I$.

c) [0,75] Obtén razonadamente A^{2016} y A^{2017}.

a) $B \cdot C^t = \begin{pmatrix} 1 & a & 1 \\ b & c & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & -2 \\ 1 & 0 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} a-2 & -4 \\ c-b+1 & -2b+2 \end{pmatrix}$

$4A = \begin{pmatrix} -4 & 4 \\ 0 & -4 \end{pmatrix}$

$\Big\} \; BC^t + 4A = \begin{pmatrix} a-6 & 0 \\ c-b+1 & -2b-2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$$\rightarrow \begin{cases} a-6=0 \longrightarrow \boxed{a=6} \\ 0=0 \checkmark \\ c-b+1=0 \longrightarrow c=b-1 \longrightarrow c=-1-1 \longrightarrow \boxed{c=-2} \\ -2b-2=0 \longrightarrow -2b=2 \rightarrow b=\frac{2}{-2} \rightarrow \boxed{b=-1} \end{cases}$$